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Fundamental Length Hypothesis and New Concept of Gauge Vector Field

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ABSTRACT

Through an analysis of quantum field theory with "fundamental length" ℓ,[1-10] a new concept of gauge vector field is determined. The electromagnetic field is considered in detail. New electromagnetic potential turns out to be 5-vector associated with De Sitter group SO(4,1). The extra fifth component, called τ-photon, similar to scalar and longitudinal photons, does not correspond to an independent dynamical degree of freedom. Gauge invariant equations of motion for all components of electromagnetic 5-potential are found. Though the new gauge group remains Abelian, it is nevertheless larger than the conventional gauge group. In particular, new gauge transformations intrinsically depend on fundamental length ℓ. Therefore one can consider them as a base for modification of QED at small distances ($\lesssim\ell$) in a profound way. The underlying physics becomes much richer due to the appearance of new interactions mediated by the τ-photons.[14]

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I. FIRST DILEMMA: FLAT OR CURVED 4-MOMENTUM SPACE?

Let us consider the free Klein-Gordon and Dirac equations in the momentum space:

$$(p^2 - m_1^2)\ \phi(p) = 0 \quad , \tag{1.1}$$

$$(\not{p} - m_2)\ \psi(p) = 0 \quad . \tag{1.2}$$

The solutions of these equations are defined on the 3-dimensional hyperboloids

$$m_1^2 - p^2 = 0\ ,\ \ m_2^2 - p^2 = 0 \quad . \tag{1.3}$$

It is important to realize that these surfaces can be equally well embedded into pseudoeuclidean (Minkowskian) 4-dimensional momentum space or into 4-dimensional De Sitter momentum space with the <u>arbitrary</u> curvature radius M

$$p_0^2 - p_1^2 - p_2^2 - p_3^2 - M^2 p_4^2 = -M^2 \quad . \tag{1.4}$$

The interacting (virtual) particles, which are located in p-space off the mass shell, are able to distinguish these two geometries if their 4-momenta are large enough:

$$p_0\ ,\ |\vec{p}|\ \gtrsim M \quad . \tag{1.5}$$

In the papers,[1-10] a new approach to the quantum field theory (QFT) has been put forward, the key idea of which can be formulated as follows:

The extrapolation off the mass shell, which is absolutely necessary for the description of an interaction in QFT, has to perform according to the rules of a geometry of De Sitter p-space (1.4).

Minkowskian geometry is the "flat limit" ($M \to \infty$) of De Sitter geometry. Therefore in the domain

$$|p_0| \;, \quad |\vec{p}| \ll M \tag{1.6}$$

the new QFT has to coincide with the conventional local QFT based on the pseudoeuclidean p-space.*

II. PSEUDOEUCLIDITY OF MOMENTUM SPACE IN A FRAMEWORK OF CONVENTIONAL QFT

Just to illustrate a pseudoeuclidian character of p-space in a framework of the local theory, let us consider QED. The gauge invariance principle requires that

$$p_\mu \to p_\mu - e_0 A_\mu \quad . \tag{2.1}$$

If $e_0 A_\mu = \text{const} \equiv k_\mu$, we have no electromagnetic field ($F_{\mu\nu} = 0$), and (2.1) reduces to the pseudoeuclidian translation of the p-space[3]

$$p_\mu \to p_\mu - k_\mu \quad . \tag{2.2}$$

Let us point out that on the surface (1.4), instead of (2.2) we should deal with De Sitter rotations in $(p_\mu p_4)$-planes:

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*References on earlier attempts to use De Sitter p-space geometry in QFT can be found in Ref. 2. The mathematical formulation of nonlocal QFT, based on ideas which are close to a concept of noneuclidean p-space, was developed by M.A. Markov many years ago.[11]

$$(p_L)' = \Lambda_L^{\ M}(k) p_M, \quad L,M = 0,1,2,3,4$$

$$\Lambda \in SO(4,1) \quad . \tag{2.3}$$

These transformations are equivalent to shifts (2.2) only in the small momentum region (1.6). Unlike (2.2), they are not a group anymore. Thus one can expect that in the new QED, based on the p-space (1.4), the gauge invariance principle and the concept of a gauge vector field (electromagnetic potential A_μ) should be revised.

III. FUNDAMENTAL MASS AND FUNDAMENTAL LENGTH

We call M the "fundamental mass." The inverse quantity

$$\ell = \frac{\hbar}{Mc} \tag{3.1}$$

will be referred to as the "fundamental length." From the above consideration it follows that the usual QFT corresponds to $\ell=0$.

Further on we shall choose the units

$$\hbar = c = \ell = M = 1 \tag{3.2}$$

to deal only with dimensionless relations. The equation (1.4) reads now:

$$p_0^{\ 2} - \vec{p}^{\,2} - p_4^{\ 2} = -1 \quad . \tag{3.3}$$

It will become clear later that a description in terms of 5-vectors is useful. Introducing the metric tensor g_{KL} of the pseudoeuclidean 5-space

$$g_{KL} = 0 \quad , \text{ if } \quad K \neq L$$

$$\text{diag } g_{KL} = (1, -1, -1, -1, -1) \quad , \tag{3.4}$$

we can write Eq. (3.3) in the form:

$$g_{KL} p^K p^L \equiv (p^2) = -1 \quad . \tag{3.5}$$

It turns out that the origin of a coordinate system in De Sitter p-space (3.3) corresponds to 5-vector

$$V^L = (0, 0, 0, 0, 1) \tag{3.6}$$

(so called "vacuum momentum"*). This is confirmed by the following observation: the "shift transformation" (2.3) with $k^L = (0, 0, 0, 0, 1)$ is equivalent to the identity transformation of SO(4,1)-group.

The De Sitterian distance S_{p0} between the arbitrary point p and the origin of the coordinate system is defined from the relation

$$\cosh S_{p0} = |g_{KL} p^K V^L| = |p^4| = |\sqrt{1 + p^2}\,| . \tag{3.7}$$

In the free particle case $p^2 = m^2$ we shall use the notation $S_{p0} = \mu$. Then (3.7) reads

$$\cosh \mu = \sqrt{1 + m^2} \tag{3.8}$$

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*This notion, in a context of QFT with the curved momentum space, has been introduced by I.E. Tamm.[2]

IV. NORMAL AND ABNORMAL EQUATIONS OF MOTION

It has been shown[4,7] that in the new scheme one has the pair equations of Klein-Gordon type:

$$2(p^4 - \cosh\mu)\phi_1 = 0 \quad , \tag{4.1a}$$

$$2(p^4 + \cosh\mu)\phi_2 = 0 \quad , \tag{4.1b}$$

where $p^4 = g^{44}p_4 = -p_4$.

Due to the relation

$$(p^2 - m^2) = (p^4 - \cosh\mu)(p^4 + \cosh\mu) \tag{4.2}$$

the standard Klein-Gordon equation (1) follows from each of Eqs. (4.1a) and (4.1b). The sign of the invariant auxiliary coordinate p^4 is a new quantum number associated with a symmetry of new p-space (3.3) under inversion

$$p^4 \to -p^4$$

and having no analog in the "flat" QFT. Let us consider in this connection the "flat" limit of Eq. (4.1), assuming naturally that the origin of coordinate system (3.6) has to belong to the small momentum region (1.6). The last statement means that a more precise definition of the pseudoeuclidean approximation, other than (1.6), is the following:

$$|p_0|, \quad |\vec{p}| \ll 1$$

$$p^4 \simeq 1 + \frac{p^2}{2} \quad . \tag{4.3}$$

With the same accuracy,

$$\cosh \mu \simeq 1 + \frac{m^2}{2} \quad . \tag{4.4}$$

Taking into account (4.3) - (4.4), one obtains from (4.1a) and (4.1b):

$$(p^2 - m^2)\phi_1 = 0$$

$$\phi_2 = 0 \quad . \tag{4.5}$$

Thus (4.1a) is a straight generalization of the conventional Klein-Gordon equation (1.1) and it will be referred to in what follows as a normal equation of motion for scalar field. Correspondingly, (4.1b) will be called an abnormal equation of motion. Let us emphasize once more that the existence of this extra equation and abnormal field ϕ_2 are intrinsically connected with the new p-space geometry, or, in other words, with our fundamental length hypothesis.

We would like to point out that due to the relations (cf. (3.5), (3.7))

$$p^4 = -(pV) \quad ,$$

$$(p^2) = -1 \quad , \quad (V^2) = -1 \quad , \tag{4.6}$$

one can write down the Eqs. (4.1a, 4.1b) in the SO(4,1)-covariant form:

$$[(p-V)^2 - 4\sinh^2\frac{\mu}{2}]\phi_1 = 0 \quad , \tag{4.7a}$$

$$[(4\sinh^2\frac{\mu}{2} - (p+V)^2]\phi_2 = 0 \quad . \tag{4.7b}$$

It is clear that for an arbitrary 5-vector A^L the following factorization formula is valid

$$(A)^2 = g_{KL}A^K A^L = (A_K\Gamma^K)(A_L\Gamma^L) \quad , \tag{4.8}$$

where $\Gamma^L = (\Gamma^0,\Gamma^1,\Gamma^2,\Gamma^3,\Gamma^4)$ denote five forth-order anticommuting matrices:

$$\{\Gamma^K,\Gamma^L\} = \Gamma^K\Gamma^L + \Gamma^L\Gamma^K = 2g^{KL}$$

$$K,L = 0,1,2,3,4 \tag{4.9}$$

Using (4.8), one can "extract the square root" from Eqs. (4.7a) and (4.7b) and obtain as a result the pair of equations, playing the role of Dirac equations in the new scheme, <u>normal</u> and <u>abnormal</u>, respectively* :[10]

$$[\not{p} - (p^4 - 1)\Gamma^4 - 2\sinh\frac{\mu}{2}]\psi_1 = 0 \quad , \tag{4.10a}$$

$$[\not{p} + (p^4 + 1)\Gamma^4 - 2\sinh\frac{\mu}{2}]\psi_2 = 0 \quad . \tag{4.10b}$$

V. SECOND DILEMMA: 4-DIMENSIONAL OR 5-DIMENSIONAL LANGUAGE?

Let us introduce now in De Sitter p-space (3.3) the orthogonal "spherical" coordinates:

$$\begin{aligned}
p^0 &= \sinh\chi & & -\infty < \chi < \infty \\
p^4 &= \cosh\chi \quad \cos\omega\ , & & 0 \le \omega \le \pi \\
p^1 &= \cosh\chi\ \sin\omega\ \sin\theta\cos\phi, & & 0 \le \theta \le \pi,\ 0 \le \phi \le 2\pi \\
p^2 &= \cosh\chi\ \sin\omega\ \sin\theta\sin\phi & & \\
p^3 &= \cosh\chi\ \sin\omega\ \cos\theta \quad . & &
\end{aligned} \tag{5.1}$$

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*The matrix Γ^4 in (4.10) is chosen in the form

$$\Gamma^4 = -i\gamma^5 = -i\begin{pmatrix} 0 & E \\ E & 0 \end{pmatrix}$$

In these coordinates the pseudoeuclidean region (4.3) is described in an especially simple way:

$$|\chi| \ll 1 \quad , \quad \omega \ll 1 \quad . \tag{5.2}$$

Having the expression (5.1) for p^0, $\vec{p}$ and p^4 , one can put them into Eqs. (4.1) and (4.10) and forget completely about the existence of 5-dimensional hyperboloid (3.3). The theory becomes manifestly 4-dimensional, though the domain of definition of the new four variables $(\chi,\omega,\theta,\phi)$ differs obviously from the domain of definition of the corresponding "flat" variables $(p^0,|\vec{p}|,\theta,\phi)$.

As an example, we write down the normal Eq. (4.1a) in terms of the four new variables:

$$2(\cosh\chi\cos\omega - \cosh\mu)\phi_1(\chi,\omega,\theta,\phi) = 0 \ . \tag{5.3}$$

In the same time one could deal with the five Cartesian coordinates $(p^0,\vec{p},p^4)$ for a description of free fields in De Sitter momentum space (3.3), if each of the equations (4.1a), (4.1b), (4.10a) and (4.10b) will be accompanied by a constraint based on Eq. (3.3). Then, for example, the analog of Eq. (5.3) will be the following set of equations:

$$\begin{cases} 2(p^4 - \cosh\mu)\phi_1(p^0,\vec{p},p^4) = 0 \\ (p_0{}^2 - \vec{p}^2 - p_4{}^2 + 1)\phi_1(p^0,\vec{p},p^4) = 0 \quad . \end{cases} \tag{5.4}$$

Similarly, for a normal spinor field ψ_1 from (4.10a) one has

$$\begin{cases} (\not{p} - (p^4 - 1)\Gamma^4 - 2\sinh\frac{\mu}{2})\psi_1(p^0,\vec{p},p^4) = 0 \\ (p_0{}^2 - \vec{p}^2 - p_4{}^2 + 1)\psi_1(p^0,\vec{p},p^4) = 0 \quad . \end{cases} \tag{5.5}$$

We should stress that both the 4-dimensional and 5-dimensional approaches are evidently equivalent to each other, and give us the same free fields. But they require an application of completely different mathematical apparatus if one considers an interaction in the gauge theory context, in the framework of these approaches.

To use the local gauge transformation, we need first to go to the x-representation in our field equations. In the 4-dimensional case (Eq. (5.3)), the approapriate Fourier transform is connected with the expansion in terms of matrix elements of the SO(4,1)-group unitary representations.[12] The new configurational x-space possesses a specific quantized structure, the size of granularities being ℓ.[2-4*] In this space Eq. (5.3) becomes the second-order differential-difference equation. The step used by the finite-difference derivative is proportional to the fundamental length ℓ.

The gauge theory which one could develop under such conditions[6] would be a covariant analog of the gauge theory on the lattice.[13]

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*In the euclidean version of the theory developed, with $p^0 \equiv i\eta$, one would obtain from Eq. (3.3):

$$\eta^2 + \vec{p}^2 + p_4{}^2 = 1 \quad . \tag{5.6}$$

The corresponding x-space is a completely discrete manifold, consisting, for example, of the Casimir operator eigenvalues of the group chain:

$$SO(5) \supset SO(4) \supset SO(3) \supset SO(2) \quad .$$

Let us come back now to the 5-dimensional formulation (Eqs. (5.4) - (5.5)). It is absolutely clear that for these pairs of equations, the appropriate Fourier transform is the expansion in terms of plane waves of the pseudoeuclidean 5-space with the metric tensor (3.4).

$$\phi(x,\tau) = \frac{1}{(2\pi)^{3/2}}\int e^{-ip_L x^L}\phi(p^0,\vec{p},p^4)\,d^5p ; L=0,1,2,3,4 \ . \qquad (5.6)$$

Here we have introduced the notation τ for the extra variable x^4:

$$x^L = (x^0,x^1,x^2,x^3,x^4) = (x^\lambda,\tau) \quad . \qquad (5.7)$$

In the configurational 5-space Eqs. (5.4) - (5.5) take the form of the set of differential equations:

$$\begin{cases} 2(-i\frac{\partial}{\partial\tau} - \cosh\mu)\phi_1(x,\tau) = 0 \\ (\Box - \frac{\partial^2}{\partial\tau^2} - 1)\ \phi_1(x,\tau) \equiv (\hat{\Box} - 1)\phi_1(x,\tau) = 0 \end{cases} \qquad (5.8)$$

and

$$\begin{cases} (i\,\Gamma^L\frac{\partial}{\partial x^L} + \Gamma^4 - 2\sinh\frac{\mu}{2})\psi_1(x,\tau) = 0 \\ (\hat{\Box} - 1)\ \psi_1(x,\tau) = 0 \quad . \end{cases} \qquad (5.9)$$

A presence of the extra variable τ in the motion equations requires that their solutions obey some additional boundary conditions. In the given (free) case such conditions can be found directly by solving Eqs. (5.8) - (5.9). From Eq. (5.8) one has:

$$\phi_1(x,\tau) = e^{i\tau\cosh\mu}\phi_1(x,0) \qquad (5.10a)$$

$$(\Box + m^2)\phi_1(x,0) = 0 \quad . \qquad (5.10b)$$

Eq. (5.10b) is the standard Klein-Gordon equation for the physical 4-dimensional scalar field with the mass equal m. Putting $\phi_1(x,0) \equiv \phi_{phys}(x)$, we have to conclude from (5.10a) that the unique boundary condition under question is of the form*:

$$\phi_1(x,\tau)\Big|_{\tau=0} = \phi_{phys}(x) \quad , \tag{5.12}$$

The analogous boundary condition can be obtained from (5.9) for the spinor field $\psi_1(x,\tau)$:

$$\psi_1(x,\tau)\Big|_{\tau=0} = \psi_{phys}(x) \quad , \tag{5.13}$$

where the field ψ_{phys} obeys the equation

$$[i\partial\!\!\!/ - (\cosh\mu - 1)\Gamma^4 - 2\sinh\frac{\mu}{2}]\psi_{phys}(x) = 0 \quad . \tag{5.14}$$

As a matter of fact the relations (5.12) - (5.13) teach us to come from 5-dimensional world back to the physical 4-world in the simplest free particle case. Taking into account the correspondence reasonings, we shall impose these boundary conditions in the interacting field case as well**.[14]

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*If $\phi_{phys}(x)$ is a hermitian field, then

$$\phi_1(x,-\tau)^{\dagger} = \phi_1(x,\tau) \tag{5.11}$$

Later on we shall consider (5.11) as the hermiticity condition in the new scheme.

**This way of eliminating the extra variable τ recalls the procedure of eliminating the relative time dependence in 4-dimensional Bethe-Salpeter equation when one is going to get the relativistic 3-dimensional quasi-potential equation by Logunov and Tavkhelidze[15] or the nonrelativistic Schrödinger equation.

Let us assume now that the set of Eqs. (5.9) describes the free electron-positron field. We would like to introduce in (5.9) the interaction with electromagnetic field A_μ, requiring the invariance of the equations considered under local gauge transformations. But all five variables $x^M = (x^\mu, \tau)$ are presented in (5.9) on an equal footing. Therefore if λ is the parameter of the global U(1)-group, then the Yang-Mills trick in the framework of the 5-dimensional approach has to look like

$$\lambda \to \lambda(x, \tau) \quad . \tag{5.14}$$

Furthermore we can conclude that the electromagnetic potential has to be 5-vector, each component being a function of the variables (x,τ):

$$A_M = (A_\mu(x,\tau),\ A_4(x,\tau));\ \begin{array}{l} M = 0,1,2,3,4 \\ \mu = 0,1,2,3 \end{array} \quad . \tag{5.15}$$

Our goal now is to find the motion equations for A_μ and A_4 which would be invariant under gauge transformations depending on $\lambda(x,\tau)$. In other words, we are looking for a generalization along our lines for the Maxwell equations*

$$\Box A_\mu(x) = \frac{\partial}{\partial x^\mu}\left(\frac{\partial A^\nu(x)}{\partial x^\nu}\right) \quad ; \tag{5.16}$$

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*Let us recall that other Maxwell equations

$$\frac{\partial F_{\mu\nu}}{\partial x^\lambda} + \frac{\partial F_{\nu\lambda}}{\partial x^\mu} + \frac{\partial F_{\lambda\mu}}{\partial x^\nu} = 0 \tag{5.18}$$

are identities due to the definition of field strengths:

$$F_{\mu\nu} = \frac{\partial A_\mu(x)}{\partial x^\nu} - \frac{\partial A_\nu(x)}{\partial x^\mu} \quad . \tag{5.19}$$

and the standard gauge transformation

$$A_\mu(x) \to A_\mu(x) - \frac{\partial\lambda(x)}{\partial x^\mu} \quad . \tag{5.17}$$

One can expect that the equations under question are a set of equations similar to (5.8) - (5.9). So we immediately can write down

$$(\hat{\Box} - 1)A_M(x,\tau) = 0 \quad , \quad M = 0,1,2,3,4 \quad . \tag{5.20}$$

The other equations for A_μ and A_4 seem to be the first order differential equations in x and τ .

VI. NEW CONCEPT OF GAUGE TRANSFORMATION

The field A_μ is a more familiar object with which to begin. In the usual approach using Lorentz gauge

$$\frac{\partial A^\nu(x)}{\partial x^\nu} = 0 \quad , \tag{6.1}$$

one would get from Eq. (5.16) the D'Alembert equation

$$\Box A_\mu(x) = 0 \tag{6.2}$$

with the constraint

$$\Box \lambda(x) = 0 \tag{6.3}$$

on the function $\lambda(x)$ in (5.17).

According to (4.1) in the new scheme, we have two analogs of Eq. (5.2). Taking into account the correspondence reasonings (see (4.5)), it is natural to choose the normal Eq. (4.1a) with $\mu=0$ as a motion equation for A_μ in the given gauge. Thus we obtain in (x,τ)-representation (the constraint (5.20) assumed to be imposed already)

$$2\left(-i\frac{\partial}{\partial\tau} - 1\right)A_\mu(x,\tau) = 0 \quad . \tag{6.4}$$

Neutrality of the "physical" field A (x,0) means (cf. (5.11)):

$$A_\mu^\dagger(x,-\tau) = A_\mu(x,\tau) \quad . \tag{6.5}$$

Equation (6.4) can be written down in the SO(4,1)-covariant form as well (see (4.7a)):

$$g^{KL}\left(i\frac{\partial}{\partial x^K} - V_K\right)\left(i\frac{\partial}{\partial x^L} - V_L\right)A_\mu(x,\tau) = 0 \quad , \tag{6.6a}$$

or

$$\Box\left[e^{i(VX)}A_\mu(x,\tau)\right] = 0 \quad . \tag{6.6b}$$

Hence, one may conclude that an analog of the constraint (6.3) on admissible $\lambda(x,\tau)$-functions, parametrizing the gauge transformation under question, is the 5-dimensional D'Alembert equation, similar to (6.6b):

$$\Box\left[e^{iVX}\lambda(x,\tau)\right] = 0 \quad . \tag{6.7}$$

It is clear now that Eq. (6.6b) is invariant under the transformation

$$e^{i(VX)}A_\mu(x,\tau) \to e^{i(VX)}A_\mu(x,\tau) - \frac{\partial}{\partial x^\mu}\left(e^{i(VX)}\lambda(x,\tau)\right) \quad . \tag{6.8}$$

Thus we have obtained a pattern of a new gauge transformation. One can now extrapolate this formula to the case of the arbitrary function $\lambda(x,\tau)$.

As we have already seen, due to the presence of spurious "vacuum vector" V^L, our approach becomes a formal SO(4,1)-covariant scheme. For this reason the 5-dimensional version of the transformation (6.8)

$$e^{i(VX)}A_M(x,\tau) \rightarrow e^{i(VX)}A_M(x,\tau) - \frac{\partial}{\partial x^M}\left(e^{(iVX)}\lambda(x,\tau)\right)$$

$$M = 0,1,2,3,4 \qquad (6.9)$$

<u>must now make sense</u>.

Taking $V^L = (0,0,0,0,1)$ one gets from (6.9):

$$A_\mu(x,\tau) \rightarrow A_\mu(x,\tau) - \frac{\partial\lambda(x,\tau)}{\partial x^\mu} \qquad (6.10a)$$

$$A_4(x,\tau) \rightarrow A_4(x,\tau) + i\lambda(x,\tau) - \frac{\partial\lambda(x,\tau)}{\partial\tau} \quad . \qquad (6.10b)$$

Let us point out that $\lambda(x,\tau)$ in (6.10) cannot be a completely arbitrary function of the five variables (x,τ). Due to Eq. (5.20), it has to obey our standard constraint:

$$(\square -1)\lambda(x,\tau) = 0 \quad . \qquad (6.11)$$

In other words, in the p-representation this function is defined in the 4-dimensional De Sitter space (3.3). This leads to an intrinsic dependence of the gauge transformation (6.10) on fundamental length. Further, due to (6.5), the function $\lambda(x,\tau)$ has to satisfy the <u>generalized hermiticity</u> condition:

$$\lambda^\dagger(x,-\tau) = \lambda(x,\tau) \quad . \qquad (6.12)$$

But if (6.12) is a new definition of hermiticity, then quantities like

$$\Omega(x,\tau) = \exp[ie_0 e^{-i\tau}\lambda(x,\tau)]; \; e_0\text{-electric charge} \qquad (6.13a)$$

have to be treated as "unitary":

$$[\Omega(x,-\tau)]^{\dagger} = \Omega^{-1}(x,\tau) \quad . \tag{6.13b}$$

In terms of (6.13a), the transformation (6.9) can be written as

$$e^{-i\tau}A_M(x,\tau) \to \Omega(x,\tau)e^{-i\tau}A_M(x,\tau)\ \Omega(x,\tau)^{-1} - \frac{1}{ie_0}\left(\frac{\partial}{\partial x^M}\Omega(x,\tau)\right)\Omega^{-1}(x,\tau) . \tag{6.14}$$

Corresponding gauge transformation of the charge particle field is of the form

$$\psi(x,\tau) \to \exp[ie_0 e^{-i\tau}\ \lambda(x,\tau)]\psi(x,\tau) \quad . \tag{6.15}$$

From (6.10b) and (6.12) one can conclude that

$$A_4(x,-\tau)^{\dagger} = -A_4(x,\tau) \quad . \tag{6.16}$$

The minus sign in (6.16) has a simple geometrical meaning: A_4 is the "fifth" component of the 5-vector A_L and changes its sign under τ-reversal.

If $\lambda(x,\tau)$ satisfies Eq. (6.7), then taking into account Eq. (6.11), one gets (cf. (6.4))

$$2(-i\frac{\partial}{\partial\tau} - 1)\lambda(x,\tau) = 0 \quad . \tag{6.17}$$

This equation obviously provides the invariance of Eq. (6.4) under the transformation (6.10a). Further taking into account Eq. (6.17), one can resolve from (6.10b) that A_4 is a gauge invariant quantity.

VII. NEW EQUATIONS FOR FREE ELECTROMAGNETIC FIELD

To find the most general equations of motion for all five components of electromagnetic potential that would be invariant under gauge transformation (6.9) - (6.10), we shall introduce the generalized field strengths:

$$F_{MN}(x,\tau) = \frac{\partial}{\partial x^N}[e^{i(VX)}A_M(x,\tau)] - \frac{\partial}{\partial x^M}[e^{i(VX)}A_N(x,\tau)] \quad . \tag{7.1}$$

For physical "vacuum vector" (3.6)

$$F_{\mu\nu} = [\frac{\partial A_\mu(x,\tau)}{\partial x^\nu} - \frac{\partial A_\nu(x,\tau)}{\partial x^\mu}]e^{-i\tau} \quad , \tag{7.2}$$

$$F_{\mu 4} = -i[A_\mu(x,\tau) + i\frac{\partial A_\mu}{\partial \tau} - i\frac{\partial A_4}{\partial x^\mu}]e^{-i\tau} \quad . \tag{7.3}$$

Let us now consider the 5-dimensional analog of Maxwell equations (5.16) and (5.18):

$$\frac{\partial F_{MN}}{\partial x_N} = 0 \tag{7.4}$$

$$\frac{\partial F_{MN}}{\partial x^L} + \frac{\partial F_{NL}}{\partial x^M} + \frac{\partial F_{LM}}{\partial x^N} = 0 \quad ; \quad M,N = 0,1,2,3,4 \quad . \tag{7.5}$$

Similar to the 4-dimensional case, Eq. (7.5) results automatically from the definition (7.1). It is also clear that Eqs. (7.4) and (7.5) are invariant under the new gauge transformation (6.9).

From (7.4) and (7.5) one obtains

$$\left\{\begin{aligned} &\frac{\partial F_{\mu\nu}(x,\tau)}{\partial x_\nu} = \frac{\partial F_{\mu 4}(x,\tau)}{\partial \tau} && (7.6a)\\ &\frac{\partial F_{\mu\nu}(x,\tau)}{\partial x^\lambda} + \frac{\partial F_{\nu\lambda}(x,\tau)}{\partial x^\mu} + \frac{\partial F_{\lambda\mu}(x,\tau)}{\partial x^\nu} = 0 && (7.6b)\\ &\frac{\partial F_{\mu\nu}(x,\tau)}{\partial \tau} = \frac{\partial F_{\mu 4}(x,\tau)}{\partial x^\nu} - \frac{\partial F_{\nu 4}(x,\tau)}{\partial x^\mu} && (7.6c)\end{aligned}\right.$$

and

$$\frac{\partial \mathcal{F}_{\mu 4}(x,\tau)}{\partial x_\mu} = 0 \quad . \tag{7.7}$$

The simplest way to derive the conventional Maxwell equations from the set (7.6) - (7.7) is to put

$$\mathcal{F}_{\mu 4}(x,\tau) = 0 \quad , \tag{7.8a}$$

or, in SO(4.1)-covariant form,

$$V^N \mathcal{F}_{MN}(x,\tau) = 0 \quad . \tag{7.8b}$$

Then from (7.6c), (7.2) and (5.19) one finds

$$\mathcal{F}_{\mu\nu}(x,\tau) = \mathcal{F}_{\mu\nu}(x,0) = F_{\mu\nu}(x) \tag{7.9}$$

and Eqs. (7.6a,b) become identical to (5.16) and (5.18).*

Let us point out that the gauge invariant condition (7.8) is in fact a first order differential equation:

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*Furthermore, taking into account (7.2 and (7.9) we can write:

$$\frac{\partial A_\mu(x,\tau)}{\partial x^\nu} - \frac{\partial A_\nu(x,\tau)}{\partial x^\mu} = e^{i\tau} F_{\mu\nu}(x) \quad .$$

Hence, this tensor is permanently satisfied the D'Alembert equation of the normal type (6.4). This equation, together with the constraint (5.20), gives rise to the equation $\Box F_{\mu\nu}(x) = 0$, which is permanently valid in the Maxwell theory of free electromagnetic field.

$$A_\mu(x,\tau) + i \frac{\partial A_\mu(x,\tau)}{\partial\tau} - i \frac{\partial A_4(x,\tau)}{\partial x^\mu} = 0 \quad . \tag{7.10}$$

It is easily seen that Eq. (7.6a) is equivalent to the following:

$$e^{-i\tau}\left[(\Box - 1)A_\mu(x,\tau) + 2\left(A_\mu(x,\tau) - i\frac{\partial A_\mu(x,\tau)}{\partial\tau} - i\frac{\partial A_4(x,\tau)}{\partial x^\mu}\right) + \right.$$
$$\left. + i\frac{\partial}{\partial x^\mu}\left(A_4(x,\tau) - i\frac{\partial A_4(x,\tau)}{\partial\tau} + i\frac{\partial A_\nu(x,\tau)}{\partial x_\nu}\right)\right] = 0. \tag{7.11}$$

Taking into account the constraint (5.20) and Eq. (7.10), one can conclude that Eq. (7.11) would be satisfied if the following first-order equation is valid:

$$A_4(x,\tau) - i\frac{\partial A_4(x,\tau)}{\partial\tau} + i\frac{\partial A_\nu(x,\tau)}{\partial x_\nu} = 0 \quad . \tag{7.12}$$

Let us note parenthetically that the constraint (5.20) for $A_4(x,\tau)$

$$(\Box - 1)A_4(x,\tau) = 0$$

is a corollary of Eq. (7.10) and Eq. (7.12).

Summarizing the results obtained, we can state that the set of differential equations

$$\left\{\begin{array}{l} 2\left[A_\mu(x,\tau) + i\dfrac{\partial A_\mu(x,\tau)}{\partial\tau} - i\dfrac{\partial A_4}{\partial x^\mu}\right] = 0 \quad , \\ 2\left[A_4(x,\tau) - i\dfrac{\partial A_4(x,\tau)}{\partial\tau} + i\dfrac{\partial A_\nu(x,\tau)}{\partial x_\nu}\right] = 0 \quad , \\ (\Box - 0)A_M(x,\tau) = 0 \end{array}\right. \tag{7.13}$$

has the following properties:

i) It is invariant under gauge transformation (6.10), where $\lambda(x,\tau)$

is obeying Eq. (6.1), and generalized hermiticity condition (6.12) ;

ii) It gives rise to the standard Maxwell equations for a free electromagnetic field.

In addition, from the explicit form of (7.13) one can conclude that

iii) It is invariant under the following transformation of 5-potential $A_M(x,\tau)$:

$$A_M(x,\tau) \rightarrow (A_\mu{}^\dagger(x,-\tau),\ -A_4{}^\dagger(x,-\tau)) \tag{7.14}$$

Hence, one has the right to put (cf. (6.5) and (6.16))

$$A_M{}^\dagger(x,-\tau) = (A_\mu(x,\tau),\ -A_4(x,\tau)) \tag{7.15}$$

Beginning now we shall consider (7.13) as equations of motion for free electromagnetic field in our approach (cf. (5.8) and (5.9)).

Let us note that the pair of first-order equations from (7.13) can be written in the Euler-Lagrange form:

$$\frac{\partial\, L\,(x,\tau)}{\partial A_M{}^\dagger(x,\tau)} = \frac{\partial}{\partial x^K}\left(\frac{\partial\, L\,(x,\tau)}{\partial\,\dfrac{\partial A_M{}^\dagger(x,\tau)}{\partial x^K}}\right) \quad ;\ K,\ M = 0,1,2,3,4 \quad , \tag{7.16}$$

where

$$\begin{aligned} L(x,\tau) = {} & 2A_M{}^\dagger(x,\tau)\ A^M(x,\tau) + i\left[A_\mu{}^\dagger(x,\tau)\frac{\partial A^\mu(x,\tau)}{\partial\tau} - \frac{\partial A_\mu{}^\dagger(x,\tau)}{\partial\tau}\,A^\mu(x,\tau)\right] + \\ & + i\left[A^4(x,\tau)\ \frac{\partial A_4{}^\dagger(x,\tau)}{\partial\tau} - A_4{}^\dagger(x,\tau)\frac{\partial A^4(x,\tau)}{\partial\tau}\right] + \\ & + 2i\left[A_\mu{}^\dagger(x,\tau)\ \frac{\partial A^4(x,\tau)}{\partial x_\mu} + A^\mu(x,\tau)\ \frac{\partial A_4{}^\dagger(x,\tau)}{\partial x^\mu}\right] \quad . \end{aligned} \tag{7.17}$$

Using (7.15), it is easy to show that

$$\mathcal{L}^{\dagger}(x,-\tau) = \mathcal{L}(x,\tau) \quad . \tag{7.18}$$

Coming back to the formulation of this theory in terms of field strengths (7.1), one can rewrite Maxwell equations for $\mathcal{F}_{\mu\nu}(x,\tau)$ in a manifestly dual-invariant form:

$$\begin{aligned} &\frac{\partial \mathcal{F}_{\mu\nu}(x,\tau)}{\partial x_{\nu}} = 0 \quad ; \quad \frac{\partial \tilde{\mathcal{F}}_{\mu\nu}(x,\tau)}{\partial x_{\nu}} \\ &\frac{\partial \mathcal{F}_{\mu\nu}(x,\tau)}{\partial \tau} = 0 \quad ; \quad \frac{\partial \tilde{\mathcal{F}}_{\mu\nu}(x,\tau)}{\partial \tau} = 0 \quad , \end{aligned} \tag{7.19}$$

where

$$\tilde{\mathcal{F}}^{\mu\nu}(x,\tau) = \tfrac{1}{2}\varepsilon^{\mu\nu\kappa\lambda}\, \mathcal{F}_{\kappa\lambda}(x,\tau) \quad . \tag{7.20}$$

The extra equation (7.8) can be considered dual-invariant also if, by definition,

$$\tilde{\mathcal{F}}^{\mu 4}(x,\tau) = \mathcal{F}^{\mu 4}(x,\tau) \quad . \tag{7.21}$$

The relations (7.20), (7.21) give rise to the following expression for the <u>dual 5-tensor</u> of electromagnetic field strengths:

$$\tilde{\mathcal{F}}^{MN}(x,\tau) = -\tfrac{1}{2}\varepsilon^{MNKLR}\, \mathcal{F}_{KL}(x,\tau)V_R + [V^M\mathcal{F}^{NK}(x,\tau) - V^N\mathcal{F}^{MK}(x,\tau)]V_K \quad , \tag{7.22}$$

where ε^{MNKLR} is a totally antisymmetric tensor such that $\varepsilon^{\mu\nu\kappa\lambda 4} = \varepsilon^{\mu\nu\kappa\lambda}$.

VIII. LORENTZ GAUGE AND τ-PHOTONS

As a first use of Eq. (7.13) one can consider them together with different gauge constraints for the 5-potential $A_M(x, \tau)$.

Choosing

$$A_4(x,\tau) = 0 \quad , \tag{8.1}$$

one evidently obtains Eq. (6.4) for $A_\mu(x,\tau)$ and the Lorentz-type gauge:

$$\frac{\partial A_\mu(x,\tau)}{\partial x_\mu} = 0 \quad . \tag{8.2}$$

According to Section VII, Eq. (8.1) is invariant under gauge transformation, which is allowed by (8.2).

Actually the constraint (8.1) is more restrictive than (8.2). Therefore it would be fairly instructive to work out the Lorentz gauge case once more starting directly with (8.2).

The motion equation for $A_4(x,\tau)$ simply becomes

$$2\left[A_4(x,\tau) - i\,\frac{\partial A_4(x,\tau)}{\partial \tau}\right] = 0 \tag{8.3}$$

and gives

$$A_4(x,\tau) = e^{-i\tau}A_4(x,0) \quad . \tag{8.4}$$

Hence, $A_4(x,\tau)$ is an <u>abnormal</u> type field (cf.(4.1b)). Furthermore, one finds

$$\Box A_4(x,0) = 0 \quad . \tag{8.5}$$

Therefore, the "physical" field $A_4(x,0)$ describes zero-mass particles.

It follows from (7.13) under the condition (8.3) that

$$\Box A_\mu(x,\tau) = 0 \quad . \tag{8.6}$$

This equation will be invariant under the gauge transformation of $A_\mu(x,\tau)$, given by (6.10a) if

$$\Box \lambda(x,\tau) = 0 \quad .$$

From here and (6.11) - (6.12) one can conclude that admissible functions $\lambda\,(x,\tau)$ have to be of the form*:

$$\lambda(x,\tau) = e^{-i\tau}\lambda_1(x) + e^{-i\tau}\lambda_2(x) \quad , \tag{8.7}$$

where

$$\Box \lambda_1(x) = \Box \lambda_2(x) = 0 \quad ; \quad \lambda_1^{\dagger}(x) = \lambda_1(x), \quad \lambda_2^{\dagger}(x) = \lambda_2(x) \quad .$$

The gauge transformation (6.10b) of A_4 can be written now as

$$A_4(x,\tau) \rightarrow A_4(x,\tau) + 2ie^{-i\tau}\lambda_2(x) \quad . \tag{8.8}$$

It is clear that Eq. (8.3) and Eq. (8.5) are invariant under this transformation.

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*According to (6.15) when $\lambda(x,\tau)$ is of the form (8.7), the gauge transformation of a charge field becomes

$$\psi(x,\tau) \rightarrow e^{ie_0\lambda_1(x)} \cdot e^{ie_0 e^{-2i\tau}\lambda_2(x)}\,\psi(x,\tau) \quad .$$

It is especially evident from here that the new gauge group is larger than conventional one, even when one is dealing with the Lorentz-gauge case.

Substituting (8.4) in the first equation of the set (7.13), one can easily find its solution.

$$A_\mu(x,\tau) = e^{i\tau} A_\mu(x,0) + \sin\tau \, \frac{\partial A_4(x,0)}{\partial x^\mu} \quad . \tag{8.9}$$

Further, applying to (8.9) the gauge transformation (6.10a) with admissible $\lambda(x,\tau)$ we obtain:

$$A_\mu(x,\tau) \to e^{i\tau}\left[A_\mu(x,0) + \frac{1}{2i}\frac{\partial A_4(x,0)}{\partial x^\mu} - \frac{\partial \lambda_1(x)}{\partial x^\mu}\right] - \\ - e^{-i\tau}\left[\frac{1}{2i}\frac{\partial A_4(x,0)}{\partial x^\mu} + \frac{\partial \lambda_2(x)}{\partial x^\mu}\right] \quad . \tag{8.10}$$

Taking into account (8.4) and (8.8), it is natural to carry out (8.10) in two steps:

i) By appropriate choice of $\lambda_2(x)$

$$\lambda_2(x) = \frac{-1}{2i} A_4(x,0) \quad ,$$

one eliminates the abnormal component $A_4(x,\tau)$. The transformation (8.10) becomes

$$A_\mu(x,\tau) \to e^{i\tau}\left[A_\mu{}'(x,0) - \frac{\partial \lambda_1(x)}{\partial x^\mu}\right] \quad , \tag{8.11}$$

where we put $A_\mu{}'(x,0) = A_\mu(x,0) - \dfrac{\partial \lambda_2(x)}{\partial x^\mu}$. In fact, we are faced with the situation considered above: electromagnetic potential is devoid of its fifth component (see (8.1) and (8.2)) and the components A_μ, depending on τ through $e^{i\tau}$, obey Eq. (6.4).

ii) By conventional choice of $\lambda_1(x)$ in (8.11), one reduces $A_\mu{}'(x,0)$ to the pure transversal form, corresponding to the radiation gauge:

$$A_0{}'(x,0) = 0 \quad ; \quad \frac{\partial \vec{A}'(x,0)}{\partial \vec{x}} = 0 \quad . \tag{8.12}$$

Thus, the free electromagnetic 5-potential $A_M(x,\tau) = (A_\mu(x,\tau), A_4(x,\tau))$, which can undergo the gauge transformation (6.10) and which satisfies Eqs.(7.13), is described by two transverse degrees of freedom, which can be referred to as the physical massless photons with two states of polarization. In other words, τ-photons, corresponding to the extra component $A_4(x,\tau)$, like the scalar and longitudinal photons, corresponding to $A_0(x,\tau)$ and $\frac{\partial \vec{A}(x,\tau)}{\partial \vec{x}}$, respectively, are pseudoparticles, which do not manifest themselves as independent dynamical degrees of freedom. However, an existence of the abnormal τ-photon component of the electromagnetic potential and the fact that a new gauge group is larger than the old one, lead to drastic consequences for symmetry properties and structure of electromagnetic interactions in the high energy domain $|p| \geq 1$.[14]

IX. CONCLUDING REMARKS

In this paper we begin a new series of publications concerning the problem of constructing a quantum field theory with a new universal constant - fundamental length ℓ. The main development after the earlier approach[1-10] is based on the following observation. The nonlocal field theory with De Sitterian momentum 4-space and granularitied space-time, described in Refs. 1-10, can be embedded in the 5-dimensional formalism manifesting familiar features of a local theory and containing the same constant ℓ as a parameter. Along these lines, one may realize that electromagnetic potential must be completed by an additional τ-photon component and be treated as a 5-vector, allowed arbitrariness in its definition being associated with a new larger gauge group.

If one would consider non-Abelian gauge theories in a framework of the given approach then the extra (fifth) components of relevant gauge vector fields have to appear. For example, in generalized QCD, we shall deal with τ-gluons. The corresponding $SU_{colour}(3)$ gauge field is

$$A_M{}^a(x,\tau) = (A_\mu{}^a(x,\tau),\ A_4{}^a(x,\tau));\quad a = 1,2,\ldots,8 \quad , \tag{9.1}$$

where the quantitites $A_4{}^a(x,\tau)$ describe the τ-gluon octet.

In complete analogy with (6.14) one can show that the gauge transformation of (9.1) is given by

$$e^{-i\tau}\,\hat{A}_M \rightarrow \Omega(x,\tau)\ e^{-i\tau}\,\hat{A}_M\,\Omega^{-1}(x,\tau) - \frac{1}{ig}\left[\frac{\partial}{\partial x^M}\ \Omega(x,\tau)\right]\Omega^{-1}(x,\tau)$$

$$\Omega(x,\tau) = \exp\,[ige^{-i\tau}\,\hat{\lambda}(x,\tau)]\ , \tag{9.2}$$

where

$$\hat{A}_M = A_M^a\ t^a \quad , \hat{\lambda} = \lambda^a(x,\tau)\ t^a,$$

$$[t^a,t^b] = i\ f^{abc}\ t^c;\ a,b,c = 1,2,\ldots,8$$

$$[\lambda^a(x,-\tau)]^+ = \lambda^a(x,\tau);\ \ (A_M^a(x,-\tau))^+ = (A_\mu^a(x,\tau),\ -A_4^a(x,\tau))\ .$$

The analog of the 5-tensor (7.1) is of the form

$$F^a_{MN} = \frac{\partial}{\partial x^N} [e^{-i\tau} A^a_M (x,\tau)] - \frac{\partial}{\partial x^M} [e^{-i\tau} A^a_N(x,\tau)] + \\ + g f^{abc} A^b_M(x,\tau) A^c_N(x,\tau) e^{-2i\tau} . \tag{9.3}$$

The further development of this formalism will be described elsewhere.

Let us stress that all "taons" (t-photons, t-gluons, etc.) are connected genetically with a curvature of p-space (3.3) or, in other words, with our fundamental length. Therefore, it is tempting to speculate that the taon field can be treated as some <u>equivalent</u> of De Sitter geometry in 4-dimensional p-space, say, in the same spirit, as the graviton field is an equivalent of space-time curvature in general relativity. Then interactions, mediated by taons, could be considered as a result of scattering of the usual particles on granularities of 4-dimensional configuration space.

Our scheme allows us to apply any language developed in a framework of gauge theory formalism. For example, one can generalize the global formulation of gauge fields [16-17] introducing an appropriate non-integrable phase factor. In the QED case this quantity can be written as*

$$\phi_{QP} = \exp \left\{ ie_o \int_P^Q e^{-i\tau} A_M(x,\tau)\, dx^M \underset{M = 0,1,2,3,4}{} \right\} , \tag{9.4}$$

---

*In usual units

$$\phi_{QP} = \exp \left\{ i \frac{e_o}{\hbar c} \int_P^Q \left[e^{-i\tau} A_\mu (x,\tau)\, dx^\mu + \ell e^{-i\tau} A_4 (x,\tau)\, d\tau \right] \right. .$$

where the integral is taken along any path connecting point P and Q in the 5-space. The loop integral

$$\oint e^{-i\tau} A_M(x,\tau)\ dx^M \ , \tag{9.5}$$

is evidently invariant under gauge transformation (6.9). Applying the 5-dimensional version of Stokes' theorem one obtains

$$\oint e^{-i\tau} A_M(x,\tau)\ dx^M = \int_S F_{MN}(x,\tau)\, d\sigma^{MN} \ , \tag{9.6}$$

where the integral on the right is to be taken over the surface S limited by the given loop. For a free field A_M ,due to Eq.(7.8a) and Eq.(7.9), one finds

$$\int_S F_{MN}(x,\tau)\ d\sigma^{MN} = \int_S F_{\mu\nu}(x)\ d\sigma^{\mu\nu} \ . \tag{9.7}$$

According to the 4-dimensional version of Stockes' theorem

$$\int_S F_{\mu\nu}(x)\ d\sigma^{\mu\nu} = \oint A_\mu(x)\ dx^\mu \ . \tag{9.8}$$

Therefore in a free case

$$\oint e^{-i\tau} A_M(x,\tau)\ dx^M = \oint A_\mu(x)\ dx^\mu \ . \tag{9.9}$$

In the presence of sources, e.g. the hypothetical Dirac monopole, we have no Eq.(9.9) and the gauge invariant loop integral (9.5) is not even a hermitian quantity. Taking into account (7.15) one can show that

$$\left[\oint e^{-i\tau} A_M(x,\tau)\ dx^M\right]^+_{\tau\to-\tau} = \oint e^{-i\tau} A_M(x,\tau)\ dx^M \ . \tag{9.10}$$

Thus the corresponding "phase factor"

$$\phi_{LOOP} = \exp \left[i\, e_o \oint e^{-i\tau} A_M(x,\tau)\, dx^M \right], \tag{9.11}$$

can be considered unitary only within the framework of the definition (6.13b). But it clearly is a factor acquired by every charged-particle wave function $\psi(x,\tau)$ after a travel around a closed loop in the presence of the given field $A_\mu(x,\tau)$:

$$\psi(x,\tau) \rightarrow \exp \left[i\, e_o \oint e^{-i\tau} A_M(x,\tau)\, dx^M \right] \psi(x,\tau) \ . \tag{9.12}$$

Let us consider a case when the integration contour lies in the (x,y)-plane. Then again applying the Stokes' theorem, one gets

$$\psi(t,\vec{x},\tau) \rightarrow \exp \left\{ i\, e_o\, e^{-i\tau} \int \left[\frac{\partial A_y(t,\vec{x},\tau)}{\partial x} - \frac{\partial A_x(t,\vec{x},\tau)}{\partial y} \right] dxdy \right\} \psi(t,\vec{x},\tau) \tag{9.13}$$

In our case this relation can be accepted as a starting point for speculations about an existence of the Dirac monopole. One can see that in order to maintain the Dirac quantization formula it is necessary to assume that the magnetic flux produced by the monopole has to depend on the variable τ only through the factor $e^{i\tau}$. This leads to additional concern about the explicit form of magnetic charge source. As a result, the Dirac quantization formula becomes slightly artificial in the given context. Therefore, one may think that if a momopole does exist, its magnetic charge g can, in principle be much smaller than the value 68.5 e_o predicted by Dirac theory.[18]

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